

Effects of Neutron Spatial Distributions on Atomic Parity Nonconservation in Cesium.

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We have examined modifications to the nuclear weak charge due to small differences between the spatial distributions of neutrons and protons in the Cs nucleus. We derive approximate formulae to estimate the value and uncertainty of this modification based only on nuclear rms neutron and proton radii. Present uncertainties in neutron distributions in Cs are difficult to quantify, but we conclude that they should not be neglected when using atomic parity nonconservation experiments as a means to test the Standard Model.

Recent measurements of transition polarizabilities [1], coupled with previous measurements of parity nonconservation (PNC) in atomic cesium [2] have significantly reduced uncertainties associated with the extraction of Q_w , the radiatively corrected weak charge of the Cs nucleus. The latest result [1], $Q_w^{\text{expt}} = -72.06(28)_{\text{expt}} (34)_{\text{atomic theory}}$ is in mild disagreement, at the 2.5σ level, with the Standard Model prediction of $Q_w^{\text{St.Mod.}} = -73.20(13)_{\text{theory}}$. [3] The experimental number requires input from atomic theory calculations [4,5] which include effects of normalization of the relevant axial electron transition matrix element in the vicinity of the nucleus. The finite nuclear size is incorporated by including $\rho_N(r)$, the spatial nuclear distribution, in the matrix elements. One possible contribution to Q_w which has been left out of the quoted numbers is the modification of the extracted weak charge due to the difference between neutron and proton spatial distributions in this nucleus with relatively large neutron excess.

The effect of the neutron distribution differing from the proton distribution in a nucleus has been explicitly considered in the atomic theory calculations, [4] and was dismissed because the estimated size was extremely small compared to existing uncertainties at the time. Other authors [6,7] have also derived and discussed this contribution further. In the case of Cs, all authors agree the effect is quite small. However, with the significant reduction in errors in recent atomic PNC measurements, the effect should no longer be neglected. As we argue below, the additional uncertainties in extracting Q_w from the data arising from neutron-proton distribution differences are slightly below the uncertainties arising from atomic theory calculations or current experimental error bars, but are comparable to Standard Model radiative correction uncertainties.

In this note, we attempt to quantify the additive contribution and uncertainties to the nuclear weak charge, Q_w , arising from the relatively poorly known spatial distribution of neutrons in the nucleus, $\rho_n(r)$. We briefly summarize some relevant nuclear structure issues, both theoretical and experimental. We also briefly discuss methods that could improve this knowledge. We present results of our numerical calculations of Q_w arising from various ρ_n distributions, and present approximate methods which show what effect differing nuclear structure model predictions would have on precision Standard Model tests.

At tree level in the Standard Model, the nuclear weak charge is $Q_w^{\text{St.Mod.}} = (1 - 4 \sin^2 \theta_W)Z - N$, with N and Z the neutron and proton number, and $\sin^2 \theta_W$ the weak mixing angle. Standard Model radiative corrections modify this formula slightly. [3] The effect of finite nuclear extent is to modify N and Z to $q_n N$ and $q_p Z$ respectively, [6] where

$$q_{n(p)} = \int f(r) \rho_{n(p)}(r) d^3 r. \quad (1)$$

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Here $f(r)$ is a folding function determined from the radial dependence of the electron axial transition matrix element inside the nucleus, and the neutron (proton) spatial distribution $\rho_{n(p)}$ is normalized to unity. It is common to characterize the neutron distribution by its rms value, R_n , since it can easily be shown that the weak charge is most sensitive to this moment. To the extent that ρ_n and ρ_p are the same, the overall nuclear size effect can be completely factored out. This has explicitly been done in the experimental extraction of Q_w . The slight difference between q_n and q_p has the effect of modifying the effective weak charge:

$$Q_w = Q_w^{\text{St.Mod}} + \Delta Q_w^{n-p}, \quad (2)$$

where ¹

$$\Delta Q_w^{n-p} = N(1 - q_n/q_p). \quad (3)$$

A naive calculation, [6] helpful for quick estimates of the effect of different possible neutron distributions on ΔQ_w^{n-p} , can be made by assuming a uniform nuclear charge distribution (zero-temperature Fermi gas), and then parameterizing the neutron distribution solely by its value of R_n . In this approximation, one solves the Dirac equation for the electron axial matrix elements, $f(r)$, near the origin by expanding in powers of α (the fine structure constant). Finally, we can assume $R_n \approx R_p$, characterizing the difference by a single small parameter, $(R_n^2/R_p^2) \equiv 1 + \epsilon$. In this case, we find [6]

$$q_p \approx 1 - (Z\alpha)^2(.26), \quad (4)$$

$$q_n \approx 1 - (Z\alpha)^2(.26 + .221\epsilon), \quad (5)$$

$$\Delta Q_w^{n-p} \approx N(Z\alpha)^2(.221\epsilon)/q_p. \quad (6)$$

Eq. 6 shows the rough dependence of the correction to the weak charge on the difference between neutron and proton distributions, characterized by ϵ . Results of this naive calculation are shown as the solid line in Fig. 1. The slope of the line demonstrates the sensitivity of the uncertainty in ΔQ_w^{n-p} to the uncertainty in rms neutron radius. The range in ϵ of ± 0.1 corresponds to a $\delta R_n/R_n$ of about 5%, which we argue below might be a reasonable estimate of the uncertainty in neutron rms radius. We do not need to rely on these approximations; we have solved the Dirac Equation numerically for s- and p- state electron wave functions given the experimental charge distribution of Cs, evaluated $f(r)$ numerically, and thus calculated q_n , q_p , and ΔQ_w^{n-p} given various model predictions for the neutron distribution. The diamonds in Fig. 1 correspond to the full calculation assuming neutron distributions with the same shape as the proton distribution, scaled to give the particular values of ϵ . The above approximations prove to be accurate, although the resulting uncertainty in ΔQ_w^{n-p} is marginally underestimated by only including the uncertainty in R_n . The additional effects of neutron distribution shape variations could slightly increase the uncertainty in the nuclear contribution to the weak charge, as demonstrated by the error bars on the diamonds in Fig. 1. These error bars arise by assuming a 2 parameter Fermi fit for the neutron distribution, $\rho_n(r) = 1/(1 + e^{[(r-c)/z]})$, and allowing the “skin thickness” parameter z_n to vary by ± 0.1 , keeping ϵ fixed. Such a variation is comparable to the difference $z_n - z_p$ in various nuclear models [18]. (A more detailed analysis of the effect of neutron shape on Q_w will be presented elsewhere [14].) From Fig 1, it is clear that the uncertainty in the radius R_n dominates the uncertainty in ΔQ_w^{n-p} .

The effect of ΔQ_w^{n-p} was understood and estimated in the atomic structure calculations [4] by first assuming $\rho_n(r) = \rho_p(r)$ (so $q_n = q_p$ and $\Delta Q_w^{n-p} = 0$) and then recalculating with a theoretical parameterization of the neutron density. [8] The resulting $\Delta Q_w^{n-p} \approx 0.06$ was extremely small, amounting to about 0.08% of the total weak charge, and was thereafter ignored. This neutron density was obtained by scaling a variational extended spherical Thomas-Fermi calculation using an effective parameterization of

¹There will be additional small multiplicative corrections to ΔQ_w^{n-p} arising from Standard Model radiative corrections, as well as additive corrections arising from e.g. internal structure of the nucleon, but these can be safely neglected since ΔQ_w^{n-p} is itself so small.

the nuclear Lagrangian (“Skyrme SkM*”), which happened to yield a nuclear neutron rms radius which differed by only 0.9% from the proton rms radius. Eq. 6 confirms the size of this shift, given only the rms neutron and proton radii. However, the assumed $\rho_n(r)$ distribution may not be an accurate representation of the correct neutron distribution. There exists both theoretical and experimental evidence that R_n might differ from R_p by significantly more than 0.9%, and thus from Eq. 6, ΔQ_w^{n-p} may be similarly underestimated.

In a more recent theoretical analysis, Chen and Vogel [7] considered two more sophisticated nuclear structure models. Both models involved a Skyrme parameterized nuclear Lagrangian, [9] computed in the spherical Hartree-Fock (HF) approximation. Such models are quite successful in predicting a wide variety of nuclear observables, including charge distributions, binding energies, bulk properties, etc. These two models (SkM* and SkIII) yielded R_n/R_p values of 1.022 and 1.016 respectively. Using the average of these values in Eq. 6 we obtain $\Delta Q_w^{n-p} = +.11$, double the estimate of Ref. [4]. Using spatial distributions of neutron and proton densities from even more recent nuclear structure models [18], we have calculated the nuclear correction directly, rather than using the approximation of Eq. 6. Using spherical Skyrme SLy4 distributions, we find $\Delta Q_w^{n-p} = +.14$. Similarly, using (spherical) Gogny distributions, including blocking, we find $\Delta Q_w^{n-p} = +.11$ (Eq. 6 for these two cases predicts +.15 and +.12 respectively). Relativistic potentials [10] typically generate significantly larger neutron radii (see discussion below), and thus would predict larger ΔQ_w^{n-p} , possibly by a factor of 2 or more, based on calculations in nearby nuclei, but no ^{133}Cs distributions for such models have been published to date. Note that if $R_n > R_p$, then ΔQ_w^{n-p} is positive. The central value of the most recent experiment [1] gives $Q_w = -72.06$, compared to $Q_w^{\text{St.Mod}} = -73.20$, so this nuclear correction is of the right sign to partially explain the small discrepancy. However, if one wanted to attribute the difference entirely to nuclear physics effects, one would require $R_n = (1.18 \pm .07)R_p$ (adding all atomic experimental and theoretical, and Standard Model theoretical errors in quadrature), which is *significantly* out of the range of any theoretical or experimental nuclear structure predictions.

The fundamental question regarding nuclear structure remains — what *uncertainty* should be associated with ΔQ_w^{n-p} ? Chen and Vogel [7] argued that a reasonable uncertainty in their calculated neutron radius might be $\delta R_n^2 \approx \pm 1 \text{ fm}^2$. According to Eq. 6, this corresponds to an uncertainty $\delta \Delta Q_w^{n-p} = \pm 0.13$. The estimate in ref. [7] for the theoretical uncertainty in Q_w for a single isotope was slightly larger, 0.25% of Q_w , i.e. ± 0.18 . This is still quite small compared to the current atomic structure uncertainty (± 0.34 in Q_w), but is as large as the uncertainty in Q_w arising from uncertainties in Standard Model radiative corrections (see Fig. 1). All of the models we have considered predict the charge radius in Cs within about 1%, but the parameter fits used to determine the Skyrme potentials are *based* in part on observables, including charge radii, in nearby semi-magic even-even nuclei. There remain various possible sources of concern that a value of $\delta R_n^2 \approx \pm 1 \text{ fm}^2$ may still be an underestimate. For example, ^{133}Cs is a deformed, odd-Z nucleus. Most nuclear structure calculations for large nuclei assume spherical symmetry with at least partially closed nuclear subshells. Pairing and blocking effects make calculations with odd N or Z less reliable, [17] as evidenced by the failure of most Skyrme HF calculations to reproduce experimentally observed “even-odd” staggering of charge radius along isotope chains. [12] In reference [7] pairing effects were included, but deformation was included only in a semi-phenomenological manner.

There exist other classes of nuclear structure models which give quite different predictions for neutron properties, for example, relativistic Hartree models based on a modified Walecka-model nuclear Lagrangian. [10] These models have seen significant improvements in recent years, and may now be viewed as competitive with more established Skyrme models in terms of their predictive power over a wide variety of observables throughout the periodic table. In a recent paper comparing models, [11] R_n^2/R_p^2 for ^{138}Ba (the nearest even-even semi-magic nucleus above Cs) ranged from 1.03 in a Skyrme model to more than 1.08 in the relativistic models. The difference in predicted R_n^2 between these two models alone exceeds 1 fm^2 . For ^{136}Xe , R_n^2/R_p^2 values vary from 1.04 to 1.09, with the predicted R_n^2 differing by well over 1 fm^2 . In another recent paper comparing models, [17] the predictions for R_n in ^{124}Sn (with a value of N/Z similar to ^{133}Cs) varied by more than 2 fm^2 between extreme models, a spread of over 8%. Again, these calculations are primarily for even-even nuclei; relativistic models have not yet been used to calculate self-consistently in the neighboring unpaired (odd Z) cases. This only *adds* to the uncertainty in the prediction of a model spread for the case of Cs. Based on these spreads, it appears

that current nuclear theory yields an uncertainty of at least 4 or 5% in R_n .

The uncertainties in neutron distributions discussed so far arise from disagreements between model predictions. It is important to note that the neutron rms radius has never been directly measured in any isotope of Cs. Indeed, it is extremely difficult to measure R_n in any nucleus - the most accurate measurements of charge radii come from electromagnetic interactions, which are dominated by the proton distribution. Elastic magnetic scattering is affected mostly by unpaired (valence) nucleons, which does not allow for a detailed or accurate measure of the bulk rms neutron radius. Data from strong interaction probes measure the “matter radius”, but are somewhat more sensitive to surface effects, and suffer from some poorly controlled systematic theoretical uncertainties arising from the models required in analyzing strong interaction observables. For example, there exist data from polarized proton elastic scattering on heavy nuclei. [13] The data are statistically of high quality, and are frequently viewed as an accurate experimental measure of R_n in several heavy nuclei, including Sn and Pb. However, the systematic uncertainties in extracting R_n , including choice of optical model and spurious variations in the result as a function of experimental beam energy, easily approach 5% or more. Other data, including pion or alpha scattering, suffer from similar uncertainties. The experimentally extracted average value from polarized proton scattering [13] and pionic atoms [19] for R_n in ^{208}Pb differ by around 3%.

Even if strong interaction measurements can be argued to provide an accurate measure of the neutron rms radius, the weak interaction is sensitive to the spatial distribution of weak charge, which can not be exactly identified with neutrons or protons, but also includes effects of other nuclear degrees of freedom including e.g. meson exchange, and is more sensitive to non-surface density variations. A parity violating electron scattering experiment [14–16] could directly measure the weak charge distribution, precisely what is needed for the interpretation of atomic PNC as a standard model test, and would be of clear value. Even if measured on another nucleus, the additional constraint on nuclear models should increase confidence in the predicted neutron distribution in Cs. As can be seen from Eq. 6, high precision atomic PNC measurements on significantly higher Z nuclei are *more* sensitive to the neutron distribution than in the case of Cs. Thus, a measurement of atomic PNC on extremely heavy nuclei might also be used as a measure of the neutron distribution, which in turn could be used to constrain the isovector parameters in the nuclear models, and thus increase the reliability of the predictions for Cs.

To summarize, ΔQ_w^{n-p} is the deviation between the experimentally extracted weak charge and Standard Model predictions due solely to differences in neutron and proton weak charge spatial distributions. The predicted value is small, typically of order 0.1, but with an uncertainty larger than the value itself, arising mostly from uncertainties in R_n . This should be compared to the nominal value of the weak charge, $Q_w^{\text{St. Mod}} = -73.20(13)$. The effect of uncertain nuclear structure is thus comparable to the present uncertainties involved in the Standard Model prediction, and for $R_n > R_p$ is of the right sign to partially explain the experimental discrepancy in Cs. For these reasons, it should be included in any future atomic PNC tests of the Standard Model. With any significant further reduction in the uncertainties in atomic theory calculations, this nuclear contribution may eventually limit the level at which Standard Model tests can be performed with atomic PNC on Cs. To reliably reduce this uncertainty would require additional direct experimental input on neutron distributions, most likely from parity violating electron scattering at low momentum transfer, e.g. at a facility such as Jefferson Lab.

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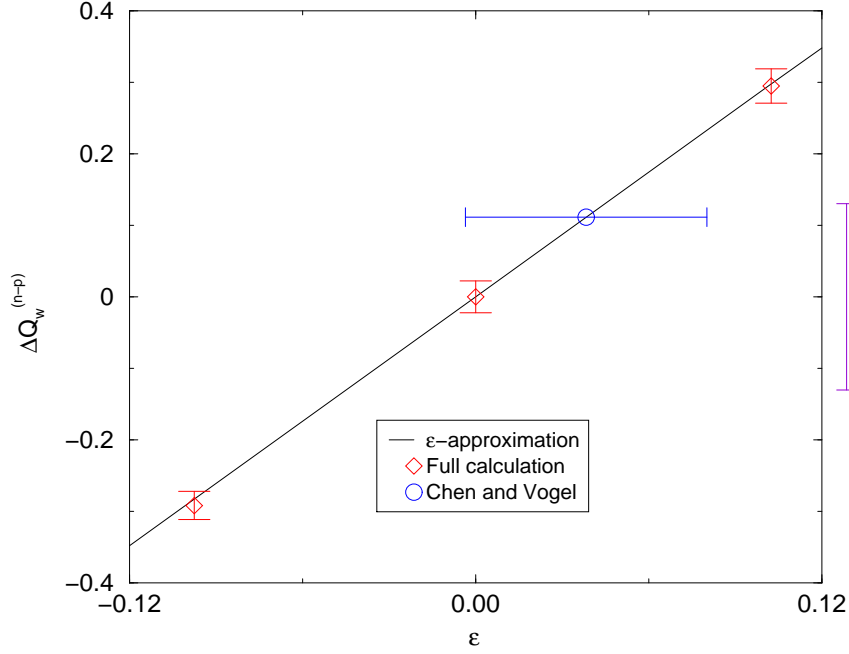


FIG. 1. Correction to the weak charge of Cs due to differences between neutron and proton spatial distributions as a function of $\epsilon \equiv R_n^2/R_p^2 - 1$. The solid curve is the prediction of Eq. 6. The diamonds are generated from a full numerical calculation with a realistic charge distribution, scaling ρ_p to get ρ_n . The error bars on the diamonds arise by varying the surface thickness of ρ_n , keeping ϵ fixed. The circle uses Eq. 6 assuming the neutron radius and uncertainty of ref. [7]. The vertical error bar to the side of the plot shows just the *uncertainty* in the Standard Model prediction [3] of Q_W .

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